ALGEBRAIC FOUNDATIONS OF PHYSICS

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Abstract. I show that the basic group-theoretical assumption of quantum mechanics is inconsistent, and has the consequence that all information about mass is lost. I show how these defects can be remedied by a simple change to the group theory, in such a way that interpretation of electron spin becomes much clearer. Once the mass information is regained, all the structure of the standard model of particle physics, including the three generations of fermions, becomes almost obvious.

1. The problem

It is well-known that the two cornerstones of 20th century physics, namely quantum mechanics and general relativity, are inconsistent with each other. In popular accounts, many eminent physicists [1, 2] are on record as saying that they believe there is an error somewhere in the foundations of quantum mechanics. Serious investigation into this problem is hampered by the fact [3] that such an opinion, expressed by someone less eminent, is likely to lead to a premature end to one’s career.

Quantum mechanics lies at the heart of the standard model of particle physics [4], which uses a great deal of group theory and representation theory [5]. Some of this group theory is quite subtle [6], and to examine it critically for errors is a job for a professional group theorist. This is a task I have been engaged in for several years, and I believe I have at last found the crucial error that is responsible for all the problems with quantum mechanics and the standard model, especially the problems of interpretation, and the measurement problem.

Group theory entered into quantum mechanics originally with the group SU(2), which was found to be necessary to model the spin of an electron. This group has a centre of order 2, consisting of the scalar matrices $\pm 1$, and central to quantum mechanics is the isomorphism

$$SU(2)/Z_2 \cong SO(3).$$

The group SO(3) is the group of rotation symmetries of Euclidean 3-space, interpreted as the real physical space around us.

2. The error

The error is to assume that this isomorphism is actually an equality. It is straightforward to prove that this is an error, because it leads quickly to a contradiction, as I shall now show. First note that Dirac [7] is generally acknowledged to have
successfully incorporated special relativity into quantum mechanics, by extending $SU(2)$ to $SL(2, \mathbb{C})$ and noting the isomorphism

\[ SL(2, \mathbb{C})/Z_2 \cong SO^+(3, 1). \tag{2} \]

Then to extend from uniform motion to accelerating motion requires extending the point group of spacetime from $SO^+(3, 1)$ to $SL(4, \mathbb{R})$. We then require a matrix group $G$ that contains $SL(2, \mathbb{C})$, such that there is an isomorphism

\[ G/Z_2 \cong SL(4, \mathbb{R}) \tag{3} \]

extending the above isomorphisms (1), (2). But there is no such matrix group $G$. This contradiction implies that the hypothesis that $SU(2)/Z_2$ is equal to $SO(3)$ is false. Yet this false assumption still lies at the heart of quantum mechanics nearly a century later. Progress cannot be made on this point, because anyone who makes such a claim is ostracised, labelled a nutter, banned from the arXiv, and their career in physics is at an end [3].

2.1. **Remark.** The standard model of particle physics contains another copy of $SU(2)$, namely the gauge group of the weak interaction, also known as the symmetry group of ‘weak isospin’. The two copies of $SU(2)$ are understood to be isomorphic but not equal, and the standard model would fall apart if one made the mistake of equating these two groups. The same therefore applies to the two copies of $SO(3)$ obtained as quotients of the spin group and isospin group. I have shown that the third copy of $SO(3)$ that is the rotation group of space (whether local or global) is distinct from the spin copy, and it is certainly generally accepted that it is distinct from the isospin copy, so there are three distinct copies of $SO(3)$ that need to be kept separate. The spin and isospin groups commute with each other in the standard model, and I see no reason to change this assumption.

3. **The correction**

Since the relationship between the spin group and spacetime cannot be the one that is used in the current theory, it must be something else. There are plenty of popular accounts [1] that explain how spin can be represented in 4-dimensional spacetime. Wikipedia has a very clear animation [8] that makes it very easy to visualise the concept of electron spin. Mathematically, then, all we have to do is extend 3-dimensional space to a non-relativistic (Euclidean) 4-dimensional spacetime, and examine the group $SO(4)$.

There are many ways to study $SO(4)$, but one of the most elegant is to use quaternions, such that $SO(4)$ is generated by left- and right-multiplications by all quaternions of norm 1. The left-multiplications form a group $SU(2)$, and the right-multiplications form another group $SU(2)$. Multiplication by $-1$ lies in both groups. The associative law for multiplication implies that the left-multiplications commute with the right-multiplications. In other words, $SO(4)$ is a central product of two copies of $SU(2)$. Let us write

\[ SO(4) = SU(2)_L \circ SU(2)_R. \tag{4} \]

If $q$ is any quaternion of norm 1, then conjugation by $q$ is the map $x \mapsto q^{-1}xq$ that fixes $x = 1$. Hence the group of all such conjugations is $SO(3)$. By mapping one-sided multiplications to conjugations we obtain natural isomorphisms

\[ SU(2)_L/Z_2 \cong SU(2)_R/Z_2 \cong SO(3). \tag{5} \]
Each of these isomorphisms expresses something important about physics. They arise from restricting the isomorphisms

\[ SO(4)/SU(2)_R \cong SO(4)/SU(2)_L \cong SO(3), \]

and therefore each isomorphism also suppresses something important.

The essential point, then, is that this mathematical foundation is capable of modelling the spin of an electron, by choosing one of the two copies of \( SU(2) \) to model spin. Then factoring out the other copy of \( SU(2) \) gives a quotient map form \( SO(4) \) onto \( SO(3) \), which restricts to the canonical projection that is required in quantum mechanics, namely

\[ SU(2) \rightarrow SO(3). \]

3.1. **Remark.** It may be possible to take the other copy of \( SU(2) \) as the isospin group. Doing so, however, would imply that the isospin \( SU(2) \) does not commute with the rotation group \( SO(3) \), so a different interpretation may be preferred.

4. À LA RECHERCHE DU TEMPS PERDU

By making this projection from \( SO(4) \) to \( SO(3) \) we have thrown away the time/energy coordinate, and we have thrown away the information that is contained in the other copy of \( SU(2) \). We have therefore thrown away a 3-parameter family of something that is important physically, but which does not appear in quantum electrodynamics. There is an obvious interpretation for this lost information as the mass parameters for the three generations of electrons. Hence the proposed new foundation for quantum mechanics actually predicts the three generations (for electrons, but not necessarily for quarks or neutrinos).

It follows that the group \( SO(4) \) contains all the information that is required to model the weak force as well as electrodynamics. A closer examination will surely allow us to explain the symmetry-breaking of the weak force, and electro-weak mixing. This takes us far from the foundations, and will be done elsewhere [9, 10].

So far, we have only considered non-relativistic quantum mechanics. To extend to special relativity, we can either extend \( SO(3) \) to \( SO(3,1) \) macroscopically, or extend \( SU(2) \) to \( SL(2,\mathbb{C}) \) quantum mechanically. Either of these extensions results in extending the group \( SO(4) \) to \( SL(4,\mathbb{R}) \), and therefore implies the other. Hence relativistic quantum mechanics must be based on the group \( SL(4,\mathbb{R}) \). There is no significant difference between special and general relativity here, as there is little difference between \( SL(4,\mathbb{R}) \) and \( GL(4,\mathbb{R}) \). The difference just consists of a scalar (mass) and a negative determinant (time reversal).

Now consider throwing away the time coordinate in \( SL(4,\mathbb{R}) \). This cannot be done via a quotient group, as was the case with \( SO(4) \), but it can be done via the subgroup \( SL(3,\mathbb{R}) \). This group tells us about relativistic quantum mechanics without a generation structure on the fermions. In other words, it is a model without mass. In the standard model, the part of the theory that ignores mass is the strong force. Hence we must use \( SL(3,\mathbb{R}) \) for the strong force. This group is similar to the standard model gauge group \( SU(3) \), as both groups are different real forms of the complex group \( SL(3,\mathbb{C}) \), and both have real dimension 8. Indeed, the whole standard model is complexified anyway, so that there is no practical distinction between these three groups.
4.1. **Remark.** It has been suggested to me that my results cannot be correct because they contradict the Coleman–Mandula theorem [11]. In fact, my results prove that the hypotheses of the Coleman–Mandula theorem do not hold, and therefore the theorem does not apply. Another way to look at this is that the Coleman–Mandula theorem implies that all parameters of the standard model are Lorentz-invariant. But this is contradicted by experiment, in particular the fact that many parameters are observed to vary with the energy scale. Hence the Coleman–Mandula theorem implies that at least one of the hypotheses is incorrect. Looked at from this point of view, the Coleman–Mandula theorem actually supports my results.

5. **Back to the future**

First, it is necessary to recover the mass information that has been lost in the standard model by throwing away the time/energy coordinate. This is largely done in [9, 10]. The former deals with invariant mass parameters, and the latter with covariant parameters and the measurement problem. This includes an explanation of how the measurement problem arises, and why it goes away once the foundations of quantum mechanics are corrected in the way I have described. Finally the principles of quantum gravity based on the above analysis are expounded in [12]. The recovered mass information resolves the dark matter problem, and has interesting consequences for the equivalence principle and the foundations of general relativity.

**References**


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